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Dynamic Loading of Structural Models by Electrostatic Forces

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Nomenclature

E = electric intensity of field
ϵ = absolute permittivity of medium
σ = charge per unit area
δ = 1 for rationalized system of units, = 4π for unrationalized system
f = forcer per unit area
C = capacitance between two conductors
Q = charge
V = potential difference between two conductors
A = area of each conductor forming capacitance C
t = separation distance between conductors forming capacitance C
F = total force
ϵ_0 = absolute permittivity of free space, i.e., a vacuum
ϵ_r = relative permittivity or dielectric constant of medium
d = separation between plates, in.
p = pressure, psi
V_p = potential difference between plates, v.

Introduction

RECENT investigations[1–3] of the buckling of shells under dynamic loading have required the development of experimental techniques for exciting various structural modes. The diamond-shaped buckling modes of an 8-in.-diam mylar cylinder were excited with an air-pressure step with a 3-msec rise time; but the desired excitation of the cylinder's 3000 cps ring mode could not be obtained with air loading. By direct application of electrostatic forces, however, the thin-walled cylinder was buckled and the loading rate was adequately high to excite the ring mode. Forcing rates greatly in excess of any structural response rates can be obtained with electrostatic loading systems. The principles of such loading, derived from the basic phenomena of the interaction of electric charges and electric fields, may be uniquely applicable in many investigations and processes.

Interaction of Electric Charges and Electric Fields

The potentials and geometric arrangement of a system of conductors separated by insulating free space or dielectrics will determine the equilibrium distribution of charge and field and therefore the distribution of forces on all surfaces. By making some of the conductors and solid dielectrics serve as the model structure, a system is produced in which the loading can be changed in the very short time required to charge a small capacitor.

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By Coulomb's theorem, the electric intensity E of the field near the surface of a conductor carrying a charge σ per unit area is [4, 5]

$$E = \delta\sigma/\epsilon \tag{1}$$

where ϵ is the absolute permittivity of the medium outside the conductor, and σ is introduced in order that its value, either 1 or 4π, will allow the use of either a rationalized or unrationalized system of units. Unless otherwise noted, all equations are dimensionally homogeneous and applicable for any consistent system of units.

The most basic parameter of interest is the electrostatic force on a conductor. The force per unit area at the surface of a conductor is[4,5]

$$f = \delta\sigma^2/2\epsilon \tag{2}$$

or, through the relationship expressed by Coulomb's theorem, it is

$$f = \epsilon E^2/2\delta \tag{3}$$

Regardless of whether σ is a positive or negative charge and regardless of the resulting direction of the vector field E, the direction of the force is always outwards from the surface, i.e., the surface of the conductor is pulled toward the dielectric; there is consequently never a push against the surface of a conductor. This suggests that the term "push-pull," sometimes applied to certain configurations of electrostatic transducers, is inappropriate (at least this terminology and the associated analyses have sometimes been misleading).

With Eqs. (2) and (3) considered as basic, it remains necessary to arrive at distributions of σ and E. Although fields of arbitrary geometric configuration can be plotted by the relaxation method, lumped-constant systems consisting of arrangements of capacitors are analytically determinate and adequate for the analysis of the loading systems usually desired.

The force per unit area on each inner surface of a parallel-plate capacitor is

$$f = \epsilon V^2/2\delta t^2 \tag{4}$$

where V is the potential difference between the two conductors and t is the thickness of the dielectric that separates the plates. Since the electrostatic force on a conductor is always outward from its surface, this force is manifest as an attraction between the plates.

The total force on each of the inner surfaces of a parallel-plate capacitor of area A is given by

$$F = \frac{\delta^2 A}{2\epsilon} = \frac{\epsilon E^2 A}{2\delta} = \frac{\epsilon V^2 A}{2\delta t^2} \tag{5}$$

when Eqs. (2–4) are multiplied by the area, and is given by

$$F = CV^2/2t = QV/2t \tag{6}$$

when the total force is desired in terms of capacitance C between the two conductors or charge Q carried by either conductor, of plate separation t, and of potential difference V.

Effect of the Dielectric

In the foregoing equations, the absolute permittivity ϵ of the dielectric is given by

$$\epsilon = \epsilon_0\epsilon_r \tag{7}$$

where ϵ_0 is the absolute permittivity of free space, i.e., in a vacuum and ϵ_r is the relative permittivity or dielectric constant of the medium separating the conductors. In an electrostatic loading system for structural models, at least part of the space between the conductors would be occupied by either a vacuum or a fluid dielectric in order to allow the structural response being investigated. If air is the fluid dielectric, the resulting ϵ is about the same as ϵ_0, since ϵ_r for air is 1.0006 at atmospheric pressure. Part of the space, however, might be filled with a sheet of solid dielectric 1) because it forms the model structure or 2) because it is necessary to increase the dielectric strength to allow the use of higher voltages in producing the desired forces.

For a parallel-plate capacitor with the gap t filled with two media of thickness t_1 and t_2 having absolute permittivities ϵ_1 and ϵ_2, respectively, the force on the conductor facing a fluid dielectric $t_1\epsilon_1$ is given by Eqs. (4–6) when t is replaced by $t_1 + (\epsilon_1/\epsilon_2)t_2$.

Applications

In designing structural models for electrostatic loading, it should be constantly recognized that conductor plates have two sides and that the outward surface forces on both sides must be determined in evaluating the net structural load. The force on the outside of a parallel-plate capacitor configuration is often negligible, however, because of the greater spacing to conductors of differing potential. But even when these forces are not negligible, the system can usually be represented accurately by mechanically and electrically connected series–parallel combinations of simple capacitors.

In the current experimental investigation, a thin mylar cylindrical shell was loaded axially and then subjected to dynamic lateral pressures by means of the two-plate electrostatic loading technique. As shown in Fig. 1, an insulated, rigid inner cylindrical surface formed one plate of a capacitor and a deposited aluminum coating on the mylar cylindrical shell formed the other plate. With the specimen plate grounded, there was no force developed on its exterior surface when the capacitor was charged to suddenly produce an inwardly directed radial force on its inner surface. This pressure step was applied by switching the specimen capacitor across a d.c. power supply as shown in Fig. 2. The magnitude of the pressure was controlled by the supply voltage in accordance with Eq. (4) and the loading rate was controlled by the series resistor inserted in the line through which the capacitor was charged.

A parallel-plate capacitor having a plate separation $t = 0.42$ cm and an area $A = 1300$ cm^2 would approximate one of the loading systems used. Using the unrationalized electrostatic c.g.s. system of units (esu), $\delta = 4\pi$, $\epsilon_0 = 1.0$, the charge density σ is in statcoulombs per square centimeter (3×10^9 statcoul = 1 coul), V is in esu of potential difference

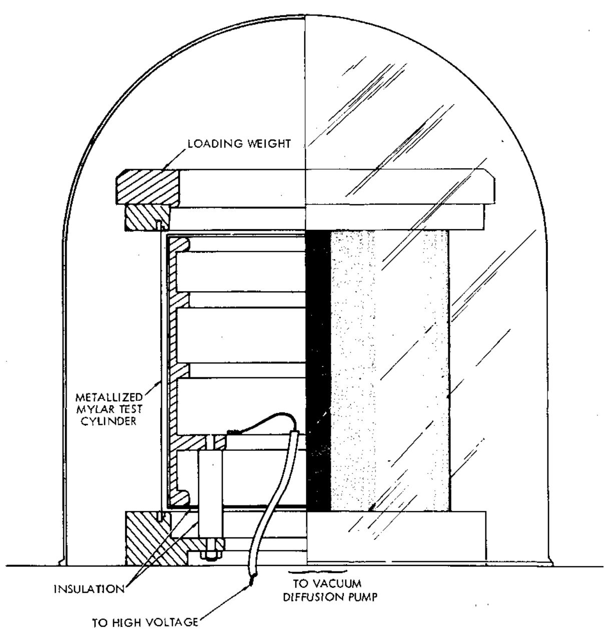

Fig. 1 Schematic cross section of two-plate electrostatic loading apparatus.

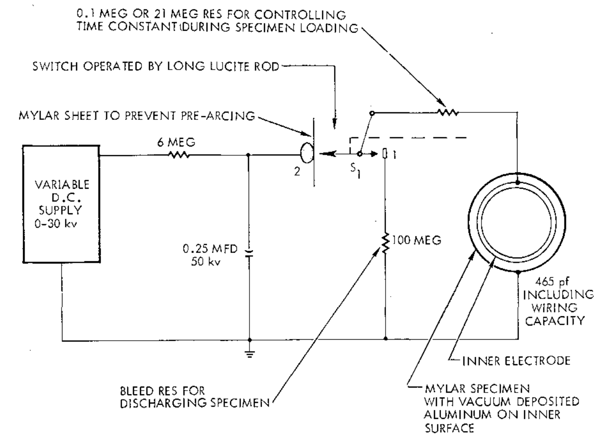

Fig. 2 Circuit for the electrostatic loading system.

(1 esu = 300 v), C is in esu of capacitance (9×10^{11} esu of capacitance = 1 farad), and the force per unit area f is in dynes per square centimeter. In a vacuum where $\epsilon = \epsilon_0 = 1.0$ and with the capacitor charged to a 20,000 v potential difference, Eq. (4) gives the pressure intensity as

$$f = \frac{\epsilon V^2}{2\delta t^2} = \frac{1.0(20{,}000/300)^2}{2 \times 4\pi \times 0.42^2} = 1000 \text{ dynes/cm}^2$$

This pressure of 0.0145 psi (68,944 dynes/cm^2 = 1 psi) can also be obtained from the expression

$$p = 0.99 \times 10^{-12}(V_p^2/d^2) \tag{8}$$

which is not dimensionally homogeneous but is applicable to a parallel-plate capacitor in air or vacuum for units of pounds per square inch, volts, and inches.

For the system illustrated in Figs. 1 and 2, the total charging capacity was measured as 465 pf giving a time constant of 46 μ sec when charged through the 0.1-meg resistor. The rapid charging time was verified on an oscillographic record of the charging current.

Conclusions

The concept of electrostatically loading a structural model for experimental studies of dynamic response has been proved feasible. The use of this system for very high rates of applying a pressure step and for applying oscillatory pressures has been demonstrated, but the method has not been fully developed.

With a controlled driving voltage, the electrostatic loading system is extremely versatile in the production of time-varying forces. By varying the electrode spacing and area, this system also offers the possibility of controlling the spatial distribution of forces applied to structural surfaces. The forces are limited, however, and attempts to improve this situation lead to problems with charges held on multilayered dielectrics. But the electrostatic loading system is capable of applying a load in a few microseconds and will operate in a vacuum, thus eliminating the effects of a heavy fluid in contact with the responding structural surface. Because of these features, consideration of the use of electrostatic forces is warranted when high-frequency modes must be excited in structural models.

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Newtonian Entropy Layer in the Vicinity of a Conical Symmetry Plane

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In their book,[1] Hayes and Probstein considered the Newtonian theory for the flow field in the vicinity of a conical-symmetry plane. It is well known[2,3] that Newtonian theory is not uniformly valid near the body surface where an entropy layer forms. In a recent analysis to be published shortly,[4] this author applied a generalization of the method of inner and outer expansions to the hypersonic flow over a general conical surface and obtained a thin shock-layer theory that is uniformly valid in the entropy layer. It is the purpose of this note to apply this general theory to the conical symmetry plane problem so as to illuminate the general nature of the entropy layer corrections to the basic Newtonian theory.

Outer Solution (Newtonian Theory)

The general conical problem is formulated in conical-curvilinear coordinates r, ξ, η, where r is the radial coordinate and ξ, η is a set of orthogonal-curvilinear coordinates on the spherical surface $r = \text{const}$ (see Fig. 1 for notation and convention). In the formal asymptotic analysis of Ref. 4, the first term of the outer expansion is completely equivalent to the usual Newtonian theory. Since the analytic properties of the Newtonian theory for general conical surfaces have been described in some detail in Ref. 5, we will not consider the general theory herein. We simply note, for later comparison with the inner solution, that the appropriate scalings for the outer region are given by $\bar{\eta} = \epsilon^{-1}\eta$, $\bar{v}_\eta = \epsilon^{-1}v_\eta/V_\infty$, $\bar{\rho} = \epsilon\rho/\rho_\infty$, where η is the coordinate, v_η is the velocity component normal to the body surface, ρ is the density, V_∞ is the freestream velocity, ρ_∞ is the freestream density, and ϵ is the small parameter that in the present problem will be set equal to the density ratio in the plane of symmetry. The radial velocity v_r, the lateral velocity v_ξ, the pressure p, and the entropy S, are $0(1)$ with respect to ϵ, and the other coordinate ξ is left unstretched.

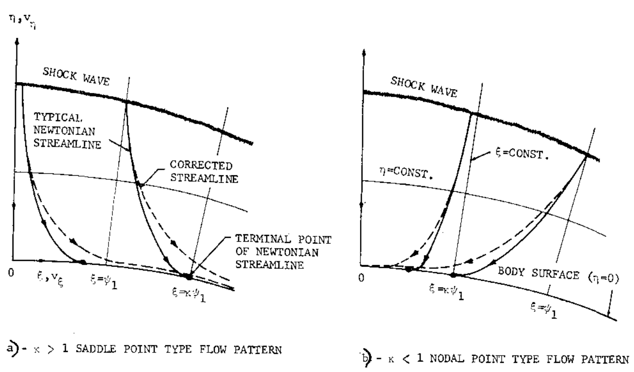

Fig. 1 Streamline geometry near a symmetry plane.

The main deviation from the Newtonian solution is centered about the solution for the crossflow streamlines. Hayes and Probstein do not carry the Newtonian solution through to obtain the equation for the streamline near the symmetry plane. However, this result is given by Eq. (6.119) in Ref. 5, and when converted to the Hayes-Probstein notation, it reads

$$\eta = \epsilon \tan\sigma_0[\kappa(\kappa - 1)^{-2}(\xi/\kappa\psi - 1 - \ln\xi/\kappa\psi) + 0(\xi^2)] \quad (1)$$

where σ_0 is the flow deflection angle in the symmetry plane and κ is the ratio of the surface curvature in the symmetry plane to the curvature of a circular cone of half angle σ_0. The crossflow stream function ψ is defined so that $\psi = \xi$ at the shock wave, and $\xi = x_1/y_1$ where x_1 and y_1 are the Cartesian coordinates appearing in the Hayes-Probstein solution. Thus,

$$\eta_s = \epsilon \tan\sigma_0 \left[\frac{\kappa \ln\kappa - (\kappa - 1)}{(\kappa - 1)^2} + 0(\xi^2)\right] \quad (2)$$

In Fig. 1 we have sketched a number of typical Newtonian streamlines for the case $\kappa > 1$ and for $\kappa < 1$. We observe that a typical streamline ψ enters the shock layer at a station $\xi = \psi$ and terminates at the body surface at station $\xi = \kappa\psi$. We note that for $\kappa > 1$, the streamline pattern has a saddle point and for $\kappa < 1$ it has a nodal-point character. For $\kappa = 1$, there is no crossflow and the streamlines lie along body normals.

The Newtonian streamlines are tangent to the body surface at the termination point and, therefore, the body surface is an envelope of Newtonian streamlines. If we also require the body surface to be a stream surface, which it must be if the lateral pressure gradient is nonzero, it is clear that the stream function ψ must be discontinuous at the body surface. It follows that S, v_r, and ρ are also discontinuous at the body surface in the Newtonian approximation.

It is important to note that the point 0 in Fig. 1 is not an isolated stagnation point within the Newtonian approximation. As a result, we find that this point is neither a branch-point nor a nodal-point singularity of the Newtonian equations. On the contrary, we find that these expected point singularities are spread out over the body surface as $\epsilon \to 0$.

Inner Solution (Entropy Layer)

In the previous paragraphs we have seen that the Newtonian solution is discontinuous at the body surface. If we try to improve the solution by constructing higher-order terms of the outer expansion, we find that these additional terms are logarithmically singular[2–4] at the body surface. A preliminary analysis indicates that this breakdown of the outer expansion is caused by the neglect of the effect of the lateral pressure gradients on the lateral component of velocity, v_ξ. This effect is accounted for in a systematic fashion by constructing a formal asymptotic expansion that, by design, is valid near the body surface.

By a relatively simple order of magnitude analysis we find that the scaled variables appropriate for the inner region are given by

$$\tilde{\rho} = \bar{\rho} \qquad \tilde{v}_\xi = \epsilon^{-1}\bar{v}_\xi = \epsilon^{-1}v_\xi/V_\infty$$

$$\tilde{v}_\eta = -\bar{v}_\eta/\bar{\eta} \qquad \tilde{\eta} = -\epsilon\ln[\bar{\eta}/F(\psi)]$$

where F is an arbitrary function introduced in order to simplify some of the final formulas. The remaining variables are unscaled with respect to ϵ. The inner solution is obtained by expanding the exact equations, written in terms of inner variables in powers of ϵ. It turns out that the pressure is constant across the inner region to all orders. The solution for the remaining variables can be reduced to quadrature

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